

Statistical Naturalness and non-Gaussianity in a Finite Universe

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We examine the behavior of n -point functions of the primordial curvature perturbations assuming our observed universe is only a subset of a larger space with statistically homogeneous and isotropic perturbations. We show that if the larger space has arbitrary correlation functions in a large family of local type non-Gaussian statistics, sufficiently biased smaller volumes will have statistics from a ‘natural’ version of that family with moments that are weakly non-Gaussian and ordered. Depending on the total size of the universe and the scale-dependence of the power spectrum, typical subsamples the size of our observed volume may be sufficiently biased to make weak non-Gaussianity whose dominant term is consistent with the usual local ansatz very likely, regardless of the statistics of the original field. We also argue that although the dominant shape of the momentum-space correlation functions may not be identical in different volumes, the characteristic behavior of the squeezed limit of the bispectrum is independent of the bias of the subsample.

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Measurements of the primordial density fluctuations are the primary tool to test the paradigm of inflationary cosmology and to distinguish between the many proposed particle physics scenarios for inflation. As our ability to test the statistics beyond the power spectrum, collectively called non-Gaussianity, becomes more advanced, new questions arise: what is the best way to test non-Gaussianity? What measurements would point definitively to particular models of inflation? So far the proposed approaches to address these questions rely either on a particle physics notion of naturalness for non-Gaussianity, either for the inflaton field [1] or for the fluctuations [2], or on mode expansions to try to capture any non-Gaussianity that is observationally accessible [3].

Here we point out a distinct and complementary way of thinking about naturalness. We suppose only that the universe is considerably larger than what we see (which is the natural outcome in many inflation models) and that there exists a homogeneous and isotropic spectrum of primordial fluctuations in the gravitational field on all scales in the entire volume. Then, for a given choice of statistics in the large volume, we can ask what statistics are typical to spatial subsets of the size of our universe. In this report, we will find a notion of statistical naturalness for typical small volumes where a family of well-behaved correlation functions is generated from a parent volume with arbitrarily fine-tuned statistics in the same family. Our results build on previous work on the non-Gaussian halo bias from the standard local ansatz [4] and in g_{NL} type non-Gaussianity [5], and can be used to more precisely characterize observable features of multi-field inflation models [6]. This work is an extension of ideas in [7] and is similar in spirit to recent results in [8–10]. We illustrate this point with a simple first example, showing that the local ansatz for non-Gaussianity, with an amplitude that is weakly non-Gaussian and whose principle term is quadratic in the underlying Gaussian, is statisti-

cally natural.

Consider a large volume characterized by side length or radius L and a smaller volume characterized by scale M . Here we will generally have in mind that M is the scale of our currently observable universe and L the scale of the entire universe (which we assume to be finite). Note that L may also just be the largest scale on which this prescription for the fluctuations is trusted. It is also sometimes useful to consider L to be the size of our observable universe and M the scale of an N-body simulation or of some local region whose Large Scale Structure we are interested in. We define the curvature perturbation in each region as the fractional shift to the scale factor a describing a background, homogeneous FRW universe:

$$a(x) = \bar{a}_L(1 + \tilde{\zeta}(x)), \quad x \in Vol_L \quad (1)$$

$$= \bar{a}_M(1 + \zeta(x)), \quad x \in Vol_M \quad (2)$$

where $|\tilde{\zeta}|, |\zeta| < 1$ by definition. We define a maximum wavenumber k_{\max} from the smallest scale we smooth over in defining the fluctuations. The average over a subsample volume M of the perturbations defined with respect to the volume L is $\langle \tilde{\zeta} \rangle_M$. These quantities are related by

$$1 + \tilde{\zeta}(x) = (1 + \langle \tilde{\zeta} \rangle_M)(1 + \zeta(x)), \quad x \in Vol_M. \quad (3)$$

The power spectrum in either volume is defined in terms of the two-point function, $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) P(k)$, and the dimensionless power spectrum is $\mathcal{P}(k) \equiv \frac{k^3}{2\pi^2} P(k)$.

Local Ansatz. Consider a simple form of the local ansatz where the curvature perturbation, $\tilde{\zeta}(x)$ is a local non-linear function of a Gaussian field $\zeta_G(x)$. In the large volume, suppose the curvature perturbation is

$$\tilde{\zeta}(x) = \tilde{N}_1 \tilde{\zeta}_G(x) + \frac{1}{2!} \tilde{N}_2 \tilde{\zeta}_G(x)^2 + \frac{1}{3!} \tilde{N}_3 \tilde{\zeta}_G(x)^3 + \dots, \quad (4)$$

where we implicitly shift the field so the mean $\langle \tilde{\zeta} \rangle$ throughout the large volume is zero. The original local ansatz [11] set $\tilde{N}_1 = 1$ and $\tilde{N}_2 = \frac{6}{5}\tilde{f}_{NL}$ to define the non-linearity parameter \tilde{f}_{NL} . We will take the \tilde{N}_i to be constants. The real-space variance of the Gaussian field $\tilde{\zeta}_G$ is

$$\tilde{\sigma}_0^2 \equiv \langle \tilde{\zeta}_G^2 \rangle = \int_{L^{-1}}^{k_{\max}} \frac{d^3k}{(2\pi)^3} \tilde{P}_G(k), \quad (5)$$

where \tilde{P}_G is the power spectrum of $\tilde{\zeta}_G$. It will also be useful to define $\tilde{\sigma}_{0l}^2$ and $\tilde{\sigma}_{0s}^2$, with limits of integration changed to (L^{-1}, M^{-1}) and (M^{-1}, k_{\max}) , respectively.

Consider $\tilde{\zeta}(x)$ for $x \in M$, a subsample. Dividing $\tilde{\zeta}_G(x \in M)$ into long and short-wavelength parts gives $\tilde{\zeta}_M = \tilde{\zeta}_{l,M} + \tilde{\zeta}_{s,M}$. Here $\tilde{\zeta}_{l,M}$ is the real space field smoothed over region M and is similar to $\langle \tilde{\zeta} \rangle_M$ defined above, up to the difference between the real space and Fourier space top-hat window functions. Following (4), this gives the local background, a constant in any particular subsample M ,

$$\tilde{\zeta}_{l,M} = \tilde{N}_1 \tilde{\sigma}_0 B + \frac{1}{2!} \tilde{N}_2 \tilde{\sigma}_0^2 B^2 + \dots, \quad (6)$$

where $B \equiv \tilde{\zeta}_{Gl,M}/\tilde{\sigma}_0$ is a measure of bias in a given subsample M . Similarly,

$$\tilde{\zeta}_s(x) = \hat{N}_1 \tilde{\zeta}_{Gs}(x) + \frac{1}{2!} \hat{N}_2 \tilde{\zeta}_{Gs}^2(x) + \frac{1}{3!} \hat{N}_3 \tilde{\zeta}_{Gs}^3(x) + \dots \quad (7)$$

contains the short-wavelength fluctuations. However, the coefficients \hat{N}_n now depend on the local background:

$$\hat{N}_n(B) = \tilde{N}_n + \tilde{N}_{n+1} \tilde{\sigma}_0 B + \frac{1}{2!} \tilde{N}_{n+2} \tilde{\sigma}_0^2 B^2 + \dots \quad (8)$$

Then the curvature perturbation in any small volume,

$$\zeta(x \in M) = \zeta_G(x) + \frac{1}{2!} N_2 \zeta_G^2(x) + \frac{1}{3!} N_3 \zeta_G^3(x) + \dots, \quad (9)$$

is related to $\tilde{\zeta}$ by $\zeta = \tilde{\zeta}_s/(1 + \tilde{\zeta}_{l,M})$, which follows from (3) and the long- and short-wavelength split above; here we have set $N_1 \equiv 1$. Reading off the Gaussian part of each expression, we have

$$\zeta_G = \frac{\hat{N}_1}{1 + \tilde{\zeta}_{l,M}} \tilde{\zeta}_{Gs}, \quad (10)$$

and consequently, the contribution \mathcal{P}_G to the power spectrum from the Gaussian part ζ_G is different in the small volume, $\mathcal{P}_G = \left(\frac{\hat{N}_1}{1 + \tilde{\zeta}_{l,M}} \right)^2 \tilde{\mathcal{P}}_G$. We also define the real-space variance of ζ_G ,

$$\sigma_0^2 \equiv \langle \zeta_G^2 \rangle = \int_{M^{-1}}^{k_{\max}} \frac{d^3k}{(2\pi)^3} P_G(k). \quad (11)$$

The N_n coefficients can be expressed in terms of the \hat{N}_n coefficients,

$$N_n(B) = \frac{(1 + \tilde{\zeta}_{l,M}(B))^{n-1}}{\hat{N}_1^n(B)} \hat{N}_n(B), \quad (12)$$

which can be verified by comparing (7) with (9) and (10). The N_n , and hence the locally averaged n -point functions, will vary among subsamples due to variation in the bias B , which is itself drawn from a Gaussian distribution with variance $\langle B^2 \rangle = \tilde{\sigma}_{0l}^2/\tilde{\sigma}_0^2 \leq 1$. In the limit $M^{-1} \rightarrow k_{\max}$, $\langle B^2 \rangle \rightarrow 1$.

The level of non-Gaussianity in $\tilde{\zeta}$ introduced by any one of the \tilde{N}_n coefficients can be quantified by $\tilde{N}_n \tilde{\sigma}_0^{n-1}$. Using (10) and (12), it is easy to show that the corresponding quantity $N_n \sigma_0^{n-1}$ for the small volume is given by

$$\lambda_n(B) \equiv N_n \sigma_0^{n-1} = \hat{N}_1^{-1} \hat{N}_n \tilde{\sigma}_{0s}^{n-1}. \quad (13)$$

The increase or decrease in the level of non-Gaussianity is determined by the same factor of \hat{N}_1^{-1} for all terms, up to higher order corrections in the \hat{N}_n , as expressed in (8). If we truncate the series at two terms, where $\tilde{N}_2 = \frac{6}{5}\tilde{f}_{NL}$, we find

$$f_{NL} \sigma_0 = \tilde{f}_{NL} \tilde{\sigma}_{0s} \left(1 + \frac{6}{5} \tilde{f}_{NL} \tilde{\sigma}_0 B \right)^{-1}. \quad (14)$$

Generically, if the series in the large volume L was a good Taylor expansion with $\tilde{N}_{n+1} \tilde{\zeta}_G < \tilde{N}_n$, the coefficients in volume M will be not too different from those in L . For unbiased subsamples, where the long wavelength modes happen to average to zero, $B = 0$ and the statistics of the subsample are identical to those of the volume L .

The running of the parameters of the series with the background bias B can also be expressed in differential form, analogous to renormalization group equations. From (8) we have $\tilde{\sigma}_0^{-1} d\hat{N}_n/dB = \hat{N}_{n+1}$, and from (6) we have $\tilde{\sigma}_0^{-1} d\tilde{\zeta}_l/dB \equiv \tilde{\sigma}_0^{-1} d\hat{N}_0/dB = \hat{N}_1$. One can show from (13) that

$$\frac{d \ln \lambda_n}{dB} = \frac{\lambda_{n+1}}{\lambda_n} - \lambda_2. \quad (15)$$

This equation is valid for any set of initial conditions $\lambda_n(0)$, that is, for any set of coefficients \tilde{N}_n , although one must take care when $B = 0$ in cases where there is no linear term in the large volume ($\tilde{N}_1 = 0$), because we have normalized the linear term to have a coefficient 1 in the small volume (9). Writing a similar differential equation for the dimensionless (connected) moments $\mathcal{M}_n \equiv \langle \zeta(x)^n \rangle_c / \langle \zeta(x)^2 \rangle^{n/2}$ would avoid that problem and be more complete, but it is also more notationally cumbersome so we do not write it here.

Weakly Non-Gaussian Ansatz. Let us now consider a case where the series in the volume L is fine tuned, with some coefficients \tilde{N}_n unusually large or small. Consider

first the case where the \tilde{N}_n with $p > n > 1$ are zero in the large volume L so that after the linear term the series starts only at order p :

$$\tilde{\zeta} = \tilde{\zeta}_G + \frac{1}{p!} \tilde{N}_p \tilde{\zeta}_G^p + \frac{1}{(p+1)!} \tilde{N}_{p+1} \tilde{\zeta}_G^{p+1} + \dots, \quad p \geq 3. \quad (16)$$

By “weakly non-Gaussian” we mean that the linear term dominates, so $\frac{1}{p!} \tilde{N}_p \tilde{\zeta}_G^{p-1/2} \ll 1$. To ensure a simple behavior of the highest moments, we also assume that the nonzero terms become smaller by the same ratio $\tilde{r} \sim (\tilde{N}_{n+1}/\tilde{N}_n) \tilde{\zeta}_G^{1/2} \ll 1$, and that $\frac{1}{p!} \tilde{N}_p \tilde{\zeta}_G^{(p-1)/2} \sim \tilde{r}^{p-1}$. This scenario gives a nearly Gaussian field in volume L whose non-Gaussian moments have some unusual properties. For $n > p$ the dimensionless moments $\tilde{\mathcal{M}}_n$ scale like $\tilde{\mathcal{M}}_n \propto \tilde{r}^{n-2}$. However, the moments with $n \leq p+1$ are not necessarily ordered (eg, $\tilde{\mathcal{M}}_n \not\propto \tilde{\mathcal{M}}_{n+1}$ is possible). Defining $\tilde{A} \equiv \tilde{\sigma}_0^2/\tilde{\zeta}_G$, with $\tilde{A}\tilde{r}^2 \ll 1$, the moments with $n \leq p$ behave as $\tilde{\mathcal{M}}_n \propto \tilde{r}^p \tilde{A}^{\frac{n}{2}}$ for $(p, n) = (\text{odd}, \text{odd})$ or $(\text{even}, \text{even})$, and $\tilde{\mathcal{M}}_n \propto \tilde{r}^{p-1} \tilde{A}^{\frac{n-1}{2}}$ for $(p, n) = (\text{even}, \text{odd})$ or $(\text{odd}, \text{even})$.

However, in subsamples the long-wavelength modes will generate the missing lower order terms:

$$\zeta = \zeta_G + \frac{1}{2!} N_2 \zeta_G^2 + \frac{1}{3!} N_3 \zeta_G^3 + \dots \quad (17)$$

With the restriction that the terms with $n > p$ in the large volume fall off according to $\tilde{r} \ll 1$, $N_n \approx \tilde{N}_p (1 + \tilde{\zeta}_l(B))^{n-1} (\tilde{\sigma}_0 B)^{p-n} / (p-n)!$ for $p \geq n > 1$, and $N_1 \equiv 1$. Interestingly, the correlation functions $\langle \zeta^n \rangle$ are not of order N_2^{n-2} but are instead dominated by the contribution from N_{n-1} .

For sufficiently biased subsamples, the series of dimensionless moments can be written, for $2 < n \leq p$,

$$\mathcal{M}_n \propto \mathcal{C} [f_{NL}^{eff} \sigma_0]^{n-2}, \quad (18)$$

where $\mathcal{C} \propto \tilde{r}^{p-1}$, and $f_{NL}^{eff} = \frac{1+\tilde{\zeta}_l(B)}{\tilde{\sigma}_0 B}$. That is, the level of non-Gaussianity and scaling of the moments in sufficiently biased subsamples is determined not by the original parameters \tilde{N}_n , but by the local background B . (In contrast, for $n > p$, $\mathcal{M}_n \propto \tilde{N}_{n-1} \sigma_0^{(n-2)/2}$.) Because $f_{NL}^{eff} \sigma_0 \simeq \sigma_0/\tilde{\sigma}_0 B$, we have $\mathcal{M}_{n+1}/\mathcal{M}_n \sim \frac{1}{B} \left(\frac{\ln(k_{\max} M)}{\ln(k_{\max} L)} \right)^{1/2}$. Consequently, for sufficiently biased subsamples, the $n \leq p$ moments will fall off as n increases. We will see that this tends to be the case for subsamples containing fewer subhorizon modes than the number of superhorizon background modes. Note also that ζ is still only weakly non-Gaussian.

Strongly Non-Gaussian Ansatz. Next, consider a case where the statistics in the volume L are very non-Gaussian:

$$\tilde{\zeta} = \frac{1}{p!} \tilde{N}_p \tilde{\zeta}_G^p + \frac{1}{(p+1)!} \tilde{N}_{p+1} \tilde{\zeta}_G^{p+1} + \dots, \quad p > 1. \quad (19)$$

where again we assume for simplicity that the first term in the series dominates. In this case the moments $\tilde{\mathcal{M}}_n$

are all of $O(1)$. In the smaller volume M the entire local ansatz series is regenerated, but with

$$N_n \approx \frac{(1 + \tilde{\zeta}_{l,M}(B))^{n-1} ((p-1)!)^n}{(\tilde{N}_p (\tilde{\sigma}_0 B)^p)^{n-1} (p-n)!}, \quad n \leq p \quad (20)$$

Now the linear term is regenerated like all the other terms, and the correlation functions $\langle \zeta^n \rangle$ are of order N_2^{n-2} ,

$$\mathcal{M}_n \propto [f_{NL}^{eff} \sigma_0]^{n-2}, \quad 2 < n \leq p \quad (21)$$

where $f_{NL}^{eff} = \frac{1+\tilde{\zeta}_{l,M}(B)}{2\tilde{N}_p (\tilde{\sigma}_0 B)^p}$. Although there is no longer an additional small factor suppressing the moments, as in (18), the scaling of the moments is otherwise the same as described above (for $n > p$, the moments again fall off with the original scale \tilde{r}). For biased enough subsamples the moments can be small and fall off rapidly; even a strongly non-Gaussian model in the large volume generates subsamples that are weakly non-Gaussian.

An easy way to see that Gaussian statistics are recovered on small scales is to consider the simple case $\tilde{\zeta} = \tilde{\zeta}_G^2$. Breaking $\tilde{\zeta}_G$ into long and short wavelength modes, we have $\tilde{\zeta} = \tilde{\zeta}_{Gl}^2 + 2\tilde{\zeta}_{Gl}\tilde{\zeta}_{Gs} + \tilde{\zeta}_{Gs}^2$. If the number of background modes is much greater than the number of short-wavelength modes, $\ln(L/M) \gg \ln(M/R)$, then as long as $\tilde{\zeta}_{Gl,M} \sim \langle \tilde{\zeta}_{Gl}^2 \rangle^{1/2}$, the linear term will be much larger than the quadratic term. In general, when the scale of the subsamples is small enough, typical subsamples will be sufficiently biased to regenerate the familiar local ansatz. (In the case of a scale-dependent power spectrum where longer wavelength modes have greater power ($n_s < 1$), the bias $B \equiv \tilde{\zeta}_{Gl,M}/\tilde{\sigma}_0$ from the background increases more rapidly as the subsample size M is decreased, causing the linear term to be boosted in size and the field ζ to be more Gaussian.)

Behavior of n -Point Function Shapes. In specifying $\tilde{\zeta}$, we determine shapes for the n -point functions on all scales. In subsamples, these shapes are still present, but (as in the two examples considered here) can be dominated by soft limits from higher n -point functions induced by the background. One might think that arbitrarily non-linear terms in $\tilde{\zeta}$ could give arbitrary k -dependence to the n -point functions. Then the usual local-shape n -point functions could be recovered in sufficiently biased small subsamples from very different shapes in the large volume. In the highly non-Gaussian case (19), the n -point functions may involve many loops (momentum space integrals), whereas in small subsamples the lower order terms allow the dominant shape to come from tree diagrams. Even for the fine-tuned nearly Gaussian case (16), it is possible for n -point functions to be dominated by contributions with many loop integrals, if we remove the earlier requirement that higher order terms fall off by $\tilde{r} \ll 1$.

To address this possibility, let us consider the p -loop contributions to the two-point function

from a given higher order term in the series: $\langle (\tilde{N}_{p+1}\tilde{\zeta}_G^{p+1})_{\mathbf{k}_1}(\tilde{N}_{p+1}\tilde{\zeta}_G^{p+1})_{\mathbf{k}_2} \rangle \in \langle \tilde{\zeta}_{\mathbf{k}_1}\tilde{\zeta}_{\mathbf{k}_2} \rangle$. This contribution can be expressed in the form

$$\tilde{P}_\zeta^{p\text{-loop}} \propto \int \prod_{i=1}^p d^3 p_i \frac{1}{|\mathbf{k} - \mathbf{p}_p|^3} \left[\prod_{i=1}^{p-1} \frac{1}{|\mathbf{p}_{i+1} - \mathbf{p}_i|^3} \right] \frac{1}{p_1^3}. \quad (22)$$

We find that after evaluating m such integrals starting from the right, with a momentum cutoff L^{-1} for all factors in denominators and taking the limit $L^{-1} \ll p_i \ll k_{\max}$, an additional factor of $\ln^m(p_{m+1}L)$ appears, giving $\tilde{P}_\zeta^{p\text{-loop}} \propto k^{-3} \ln^p(kL)$. Additional terms are also introduced, but either have weaker momentum dependence or can be discarded in the limit $p_i/k_{\max} \ll 1$. The appearance of the scale L in these expressions should not be interpreted as measurability of L , since its value is completely degenerate with the amplitude of the power spectrum, the spectral index, and analogous quantities for higher order correlation functions (see, eg, [12]).

This analysis can be generalized to the three-point and higher n -point functions; an n -loop contribution to the bispectrum will involve terms of the form [13]

$$\frac{1}{k_1^3 k_2^3} \ln^{m_1}(k_1 L) \ln^{m_2}(k_2 L) \ln^{m_3}(\min(k_1, k_2)L) + \text{perms.}, \quad (23)$$

where $\sum m_i = n$ and $m_{1,2,3}$ are the number of loops coming from contractions between different pairs among three terms in the series contributing to the bispectrum. In the squeezed limit, $k_1 \rightarrow 0$ and $k_2 \simeq k_3$, only terms with $m_1 = 0$ will contribute, so the squeezed limit will still be characterized by the usual k_1^{-3} dependence.

In conclusion, then, a local ansatz (4) with arbitrarily fine-tuned coefficients \tilde{N}_n can contribute additional logarithmic k -dependence to n -point functions, but the behavior in the squeezed limit remains unchanged; this characteristic behavior of the local model cannot be erased by fine-tuning the coefficients.

The question of shape is also more complex for higher n -point functions in that there are more tree level shapes. For the local model, there are two trispectrum shapes typically discussed: $T_g = g_{NL} P_G(k_1) P_G(k_2) P_G(k_3)$ and $T_\tau = \tau_{NL} P_G(k_1) P_G(k_2) P_G(|\mathbf{k}_1 + \mathbf{k}_3|)$, with sums over permutations; in our case $\tau_{NL} = (\frac{6}{5} f_{NL})^2$. For the nearly Gaussian ansatz (16), cubic and quadratic terms will be regenerated, with $g_{NL}/f_{NL}^2 \sim \tilde{r}^{-(p-1)} \gg 1$, so the T_g shape will dominate the T_τ shape in sufficiently biased subsamples. In the large volume this is also true; the leading term $\langle \tilde{\zeta}_{G,\mathbf{k}_1} \tilde{\zeta}_{G,\mathbf{k}_2} \tilde{\zeta}_{G,\mathbf{k}_3} (\tilde{N}_p \tilde{\zeta}^p)_{\mathbf{k}_4} \rangle$ (or $p+1$ for even p) has the same momentum dependence.

For the highly non-Gaussian ansatz (19), in sufficiently biased subsamples the quadratic and cubic terms are large and $g_{NL} = O(f_{NL}^2)$ (assuming $p > 2$), so the two shapes contribute equally. In the large volume this is also true because the loop integrals in the trispectrum can be contracted diagrammatically in different ways, contribut-

ing terms that approximate both tree level shapes [13]. As an exception, for $p = 2$ the τ_{NL} shape dominates in both volumes because the quadratic term is abnormally large compared to the cubic term.

This generalizes to higher n -point functions as well: the tree-level shape(s) that are dominant throughout the large volume will also dominate in sufficiently biased small subsamples. For (16), the shape from $\langle \tilde{\zeta}_{G,\mathbf{k}_1} \times \dots \tilde{\zeta}_{G,\mathbf{k}_{n-1}} (\tilde{N}_{n-1} \tilde{\zeta}^{n-1})_{\mathbf{k}_n} \rangle$ will dominate for any n -point function on all scales; for (19), contributions to n -point functions from $\zeta_G^{m \leq p}$ terms (the regenerated missing terms) will dominate, and the momentum dependence in the large volume will be similar.

Conclusion. From these examples we conclude that for local non-Gaussianity a weakly non-Gaussian series, with terms falling off by a characteristic ratio r and the moments following a hierarchical scaling $\mathcal{M}_{n+1}/\mathcal{M}_n \sim r$, is statistically natural: such a series is recovered in typical subsamples on sufficiently small scales from an arbitrary set of local terms in the large volume. Any model for local non-Gaussianity can be expressed as a superposition of the two specific cases considered here.

These results suggest two important things for understanding what limits on or detection of non-Gaussianity imply for theories of the primordial universe. First, the form of the local ansatz is protected against changes of scale: although the finiteness of the observable universe means a one-to-one map between observations and theory parameters may not be possible, subsampling does not lead to correlation functions with arbitrary shape in momentum space. Second, these results are independent of a specific dynamical origin for the fluctuations and suggest that purely statistical arguments could be used to define a space of most plausible non-Gaussian models to be tested against observations. Our results are complementary to other statistical restrictions on the relative size of certain moments [14]. Extensions and applications of this result for local, scale-dependent local and non-local non-Gaussianity are in progress.

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